

6669/01: Further Pure Mathematics FP3

Question number	Scheme	Marks
1.	$\begin{vmatrix} (7-\lambda) & 6 \\ 6 & (2-\lambda) \end{vmatrix} = 0$ $(7-\lambda)(2-\lambda) - 36 = 0$ $\lambda^2 - 9\lambda + 14 - 36 = 0$ $\lambda^2 - 9\lambda - 22 = 0$ $(\lambda - 11)(\lambda + 2) = 0 \Rightarrow \lambda_1 = -2, \lambda_2 = 11$	<p>M1 A1</p> <p>M1 A1 (4)</p>
2.	$9\left(\frac{e^x + e^{-x}}{2}\right) - 6\left(\frac{e^x - e^{-x}}{2}\right) = 7$ $3e^{2x} - 14e^x + 15 = 0$ $(3e^x - 5)(e^x - 3) = 0 \quad e^x = \frac{5}{3}, \quad e^x = 3$ $x = \ln \frac{5}{3} \quad x = \ln 3$	<p>B1</p> <p>M1 A1</p> <p>M1 A1</p> <p>A1 (6)</p>
3.	$s = \int_0^{2\pi} \left[\dot{x}^2 + \dot{y}^2 \right]^{\frac{1}{2}} dt$ $\frac{dx}{dt} = \dot{x} = a(1 - \cos t); \quad \frac{dy}{dt} = \dot{y} = a \sin t$ $s = \int_0^{2\pi} a \left[(1 - \cos t)^2 + \sin^2 t \right]^{\frac{1}{2}} dt = a \int_0^{2\pi} [2 - 2\cos t]^{\frac{1}{2}} dt$ $= 2a \int_0^{\frac{\pi}{2}} \sin\left(\frac{t}{2}\right) dt, = -4a \left[\cos\left(\frac{t}{2}\right) \right]_0^{2\pi} = 8a$	<p>M1 A1;</p> <p>A1</p> <p>M1 A1</p> <p>A1ft (6)</p>

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4.	$x = 2 \sinh t$ $\sqrt{(x^2 + 4)} = (4 \sinh^2 t + 4)^{\frac{1}{2}} = 2 \cosh t$ $dx = 2 \cosh t dt$ $I = \int \sqrt{(x^2 + 4)} dx = 4 \int \cosh^2 t dt$ $= 2 \int (\cosh 2t + 1) dt$ $= \sinh 2t + 2t + c$ $= \frac{1}{2} x \sqrt{(x^2 + 4)} + 2 \operatorname{arsinh} \left(\frac{x}{2} \right) + c$	B1 M1A1 M1 A1 M1 A1ft (7)
5.	<p>(a)</p> $y = \arcsin x$ $\Rightarrow \sin y = x$ $\cos y \frac{dy}{dx} = 1$ $\frac{dy}{dx} = \frac{1}{\cos y} = \frac{1}{\sqrt{1-x^2}}$ <p>(b)</p> $\frac{d^2 y}{dx^2} = -\frac{1}{2} (1-x^2)^{-\frac{3}{2}} (-2x)$ $= x (1-x^2)^{-\frac{3}{2}}$ $(1-x^2) \frac{d^2 y}{dx^2} - x \frac{dy}{dx} = (1-x^2) x (1-x^2)^{-\frac{3}{2}} - x (1-x^2)^{-\frac{1}{2}} = 0$	M1 M1 A1 (3) M1 A1 M1 A1 (4)

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6. (a)	$I_n = \int_0^{\frac{\pi}{2}} x^n \sin x \, dx$ $= \left[x^n (-\cos x) \right]_0^{\frac{\pi}{2}} - \int_0^{\frac{\pi}{2}} n x^{n-1} (-\cos x) \, dx$ $= 0 + n \left\{ \left[x^{n-1} \sin x \right]_0^{\frac{\pi}{2}} - \int_0^{\frac{\pi}{2}} (n-1) x^{n-2} \sin x \, dx \right\}$ $= n \left[\left(\frac{\pi}{2} \right)^{n-1} - (n-1) I_{n-2} \right]$ <p>So $I_n = n \left(\frac{\pi}{2} \right)^{n-1} - n(n-1) I_{n-2}$</p>	<p>M1 A1</p> <p>A1</p> <p>A1 (4)</p>
(b)	$I_3 = 3 \left(\frac{\pi}{2} \right)^2 - 3.2 I_1$ $I_1 = \int_0^{\frac{\pi}{2}} x \sin x \, dx = \left[x(-\cos x) \right]_0^{\frac{\pi}{2}} + \int_0^{\frac{\pi}{2}} \cos x \, dx$ $= \left[\sin x \right]_0^{\frac{\pi}{2}} = 1$ $I_3 = 3 \left(\frac{\pi}{2} \right)^2 - 6 = \frac{3\pi^2}{4} - 6$	<p>M1</p> <p>A1</p> <p>M1 A1 (4)</p>

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7. (a)	$\mathbf{A}(x) = \begin{pmatrix} 1 & x & -1 \\ 3 & 0 & 2 \\ 1 & 1 & 0 \end{pmatrix}$ Cofactors $\begin{pmatrix} -2 & 2 & 3 \\ -1 & 1 & x-1 \\ 2x & -5 & -3x \end{pmatrix}$ Determinant = $2x - 3 - 2 = 2x - 5$ $\mathbf{A}^{-1}(x) = \frac{1}{2x-5} \begin{pmatrix} -2 & -1 & 2x \\ 2 & 1 & -5 \\ 3 & (x-1) & -3x \end{pmatrix}$	 M1 A1 A1 A1 M1 A1 M1 A1ft (8)
(b)	$\begin{pmatrix} p \\ q \\ r \end{pmatrix} = \mathbf{B}^{-1} \begin{pmatrix} 2 \\ 3 \\ 4 \end{pmatrix} = \frac{1}{1} \begin{pmatrix} -2 & -1 & 6 \\ 2 & 1 & -5 \\ 3 & 2 & -9 \end{pmatrix} \begin{pmatrix} 2 \\ 3 \\ 4 \end{pmatrix}$ $= (17, -13, -24)$	 M1 A1ft M1 A1 (4)

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8. (a)	$\overrightarrow{AB} = (-1, 3, -1); \overrightarrow{AC} = (-1, 3, 1).$ $\overrightarrow{AB} \times \overrightarrow{AC} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -1 & 3 & -1 \\ -1 & 3 & 1 \end{vmatrix}$ $= \mathbf{i}(3+3) + \mathbf{j}(1+1) + \mathbf{k}(-3+3)$ $= 6\mathbf{i} + 2\mathbf{j}$ <p>Area of $\Delta ABC = \frac{1}{2} \overrightarrow{AB} \times \overrightarrow{AC}$</p> $= \frac{1}{2} \sqrt{36+4} = \sqrt{10} \text{ square units}$	M1 A1 M1 A1 A1 M1 A1ft (7)
(b)	<p>Volume of tetrahedron = $\frac{1}{6} \overrightarrow{AD} \cdot (\overrightarrow{AB} \times \overrightarrow{AC})$</p> $= \frac{1}{6} -12+8 $ $= \frac{2}{3} \text{ cubic units}$	M1 A1 (2)
(c)	<p>Unit vector in direction $\overrightarrow{AB} \times \overrightarrow{AC}$ i.e. perpendicular to plane containing A, B, and C is</p> $\mathbf{n} = \frac{1}{\sqrt{40}} (6\mathbf{i} + 2\mathbf{j}) = \frac{1}{\sqrt{10}} (3\mathbf{i} + \mathbf{j})$ $p = \mathbf{n} \cdot \overrightarrow{AD} = \frac{1}{\sqrt{10}} (3\mathbf{i} + \mathbf{j}) \cdot (-2\mathbf{i} + 4\mathbf{j}) $ $= \frac{1}{\sqrt{10}} -6+4 = \frac{2}{\sqrt{10}} \text{ units.}$	M1 M1 A1 (3)

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9. (a)	$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ $\frac{2x}{a^2} - \frac{2y}{b^2} \frac{dy}{dx} = 0$ $\frac{dy}{dx} = \frac{2x}{a^2} \frac{b^2}{2y} = \frac{b^2}{a^2} \frac{a \sec \theta}{b \tan \theta} = \frac{b}{a \sin \theta}$ <p>Gradient of normal is then $-\frac{a}{b} \sin \theta$</p> <p>Equation of normal: $(y - b \tan \theta) = -\frac{a}{b} \sin \theta (x - a \sec \theta)$</p> $ax \sin \theta + by = (a^2 + b^2) \tan \theta$	<p>M1 A1</p> <p>M1 A1</p> <p>M1 A1 (6)</p>
(b)	<p>M: A normal cuts $x = 0$ at $y = \frac{(a^2 + b^2)}{b} \tan \theta$</p> <p>B normal cuts $y = 0$ at $x = \frac{a^2 + b^2}{a \sin \theta} \tan \theta$</p> $= \frac{(a^2 + b^2)}{a \cos \theta}$ <p>Hence M is $\left[\frac{(a^2 + b^2)}{2a} \sec \theta, \frac{(a^2 + b^2)}{2b} \tan \theta \right]$</p> <p>Eliminating θ</p> $\sec^2 \theta = 1 + \tan^2 \theta$ $\left[\frac{2aX}{a^2 + b^2} \right]^2 = 1 + \left[\frac{2bY}{a^2 + b^2} \right]^2$ $4a^2 X^2 - 4b^2 Y^2 = [a^2 + b^2]^2$	<p>M1 A1</p> <p>A1</p> <p>M1</p> <p>M1</p> <p>A1</p> <p>A1 (7)</p>